Where does the Rho Go? Chirally Symmetric Vector Mesons in the Quark–Gluon Plasma.

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If the phase transition of QCD at nonzero temperature is dominated by the (approximate) restoration of chiral symmetry, then the transition might be characterized using a gauged linear sigma model. Assuming that vector meson dominance holds, such sigma models predict that at the temperature of chiral restoration, the pole mass of the thermal ρ meson is greater than that at zero temperature; in the chiral limit and in weak coupling this mass is $\sim 962\, MeV$. The width of the thermal $\rho - a_1$ peak is estimated to be about $200 - 250\, MeV$.

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When the quark–gluon plasma is produced by the collision of large nuclei at ultrarelativistic energies, such as at $RHIC$ and LHC, the crucial question is how to detect its presence as the plasma expands and cools into ordinary hadronic matter. A promising signal is to look at the production of dileptons, since they escape from the fireball essentially without interaction. The most prominent feature of the dilepton spectra are the peaks from their coupling to vector mesons.

Vector mesons can be classified into two types. For mesons such as the ρ, their lifetime is so short that they decay within the plasma. Consequently, the shift in their mass and width from interactions in the plasma — the nature of the "thermal ρ" — are in principle observable [1]. The second type of mesons are those whose lifetime is so long that they decay outside of the plasma, like the J/Ψ. Then any shift in the mass or width is not observable, but one can measure a relative depletion in the height of the peak [2].

In this Letter I investigate the nature of the thermal ρ within the context of a gauged linear sigma model [3]. Several other authors have conducted similar studies in these [4], [5] and other [6]-[12] models. The principal point herein is that at least in weak coupling, a general feature of gauged linear sigma models is that at the point where chiral symmetry is restored, the mass of the thermal ρ is *greater* than that at zero temperature. The shift in the ρ mass can be relatively large, on the order of T_χ, where T_χ is the temperature for the restoration of chiral symmetry.

I work with two flavors, assuming that the effects of the axial anomaly are always large, so the global chiral symmetry is $SU(2)_l \times SU(2)_r$. Introducing the matrices $t^0 = \mathbf{1}/2$ and t^a, $tr(t^a t^b) = \delta^{ab}/2$, the scalar field Φ is

$$\Phi = \sigma\, t^0 + i\vec{\pi}\cdot\vec{t}\, ;$$

$\vec{\pi}$ is the $J^P = 0^-$ isotriplet pion field and σ a 0^+ isosinglet field. For the left and right handed vector fields I take

$$A^\mu_{l,r} = (\omega^\mu \pm f_1^\mu) t^0 + (\vec{\rho}^\mu \pm \vec{a}_1^\mu)\cdot\vec{t}$$

where ω and $\vec{\rho}$ are 1^- fields, and f_1 and $\vec{a}_1$ are 1^+ fields. According to the principle of vector meson dominance [3], the dimensionless couplings of the vector fields to themselves and to Φ are exclusively those which follow by promoting the global chiral symmetry to a local symmetry. Introducing the coupling constant g for vector meson dominance, the appropriate covariant derivative and field strengths are $D^\mu\Phi = \partial^\mu\Phi - ig(A^\mu_l\Phi - \Phi A^\mu_r)$ and $F^{\mu\nu}_{l,r} = \partial^\mu A^\nu_{l,r} - \partial^\nu A^\mu_{l,r} - ig[A^\mu_{l,r}, A^\nu_{l,r}]$. The effective lagrangian is then

$$\mathcal{L} = tr\left(|D^\mu\Phi|^2 - \mu^2|\Phi|^2 + \lambda(|\Phi|^2)^2 - 2ht^0\Phi \right.$$
$$\left. + \frac{1}{2}(F^{\mu\nu}_l)^2 + \frac{1}{2}(F^{\mu\nu}_r)^2 + m^2\left((A^\mu_l)^2 + (A^\mu_r)^2\right)\right)\ . \quad (1)$$

Including g, the parameters of the model are a mass squared $-\mu^2$, which drives spontaneous symmetry breaking at zero temperature, a dimensionless scalar coupling λ, a background field h to make the pions massive, and a mass term $\sim m^2$ for the gauge fields [13]. Much of the physics of this lagrangian can be understood from the kinetic term for the scalar field,

$$tr\left(|D^\mu\Phi|^2\right) = \frac{1}{2}\left((\partial^\mu\sigma + g\vec{a}_1^\mu\cdot\vec{\pi})^2 \right.$$
$$\left. + \left(\partial^\mu\vec{\pi} + g\vec{\rho}^\mu\times\vec{\pi} - g\vec{a}_1^\mu\sigma\right)^2 + g^2\left(\sigma^2 + \vec{\pi}^2\right)(f_1^\mu)^2\right) \quad (2)$$

Because it couples to the (isosinglet) current for fermion number, the ω^μ field drops completely out of (2). There are interactions of ω^μ due to effects of the anomaly, but these are neglected in this work.

I stress how remarkable the principle of vector meson dominance is. If one constructs the most general lagragian consonant with the *global* chiral symmetry of $SU(2)_l \times SU(2)_r$, then instead of a one coupling constant g, many more dimensionless coupling constants are required [14]. Vector meson dominance limits the breaking of the local chiral symmetry solely to soft mass terms [13], such as that $\sim m^2$ in (1). As I discuss at the end of this Letter, if the principle of vector meson dominance is abandoned, then very different predictions follow.

Of course the price paid is that the theory is not perturbatively renormalizable. For a vector field with mass m, in momentum space the propagator is $\Delta^{\mu\nu}(P) = (\delta^{\mu\nu} - P^\mu P^\nu/m^2)/(P^2+m^2)$, which is ~ 1 and so badly behaved at large P. In the present analysis this lack of renormalizability is inconsequential. This is because I assume that I am always in a regime where the temperature $T \leq T_\chi \ll m$, and for such low temperatures the effects of quantum vector fields should be temperature independent.

When spontaneous symmetry breaking occurs, so $\sigma \to \sigma_0 + \sigma$, the vector meson masses are [13]

$$m_\rho^2 = m_\omega^2 = m^2 , \;\; m_{a_1}^2 = m_{f_1}^2 = m^2 + (g\sigma_0)^2 , \qquad (3)$$

Further, from (2) $\sigma_0 \neq 0$ generates a mixing between the $\vec{a}_1^\mu$ field with $\partial^\mu\vec{\pi}$. This produces a type of "partial" Higgs effect, whereby the standard results in a linear sigma model are modified by ratios of m_{a_1}/m_ρ:

$$f_\pi = \frac{m_\rho}{m_{a_1}}\sigma_0 , \;\; m_\pi^2 = \frac{m_{a_1}^2}{m_\rho^2}\frac{h}{\sigma_0} , \;\; m_\sigma^2 = \frac{h}{\sigma_0} + 2\lambda\sigma_0^2 . \quad (4)$$

In MeV I use the values $f_\pi = 93$, $m_\pi = 137$, $m_\rho = 770$, and $m_{a_1} = 1260$. Notice that the value of ratio $m_{a_1}/m_\rho \sim 1.6$ is significantly larger than one. These values determine $\sigma_0 = 152\,MeV$, $g = 6.55$, $h = (102\,MeV)^3$, and $m = 770\,MeV$. The values of the remaining parameters depend upon the value of m_σ. I choose two representative values [15]; $m_\sigma = 600\,MeV$ gives $\lambda = 7.62$ and $\mu = 412\,MeV$, while $m_\sigma = 1000\,MeV$ gives $\lambda = 21.4$ and $\mu = 700\,MeV$. With these values of λ and g the theory is manifestly in a strong coupling regime. Nevertheless, to gain a qualitative understanding of the physics I work to lowest order in a loop expansion.

In weak coupling it is easy to compute the thermal masses at the temperature of chiral symmetry restoration, T_χ. For simplicity I work in the chiral limit, $h = 0$, where $T_\chi^2 = 2\sigma_0^2$, so $T_\chi = 215\,MeV$ [16], [17]. At T_χ I can compute in the symmetric phase, working from above. A technical but crucial point is that it is necessary to compute the self energies not at zero momentum, but on the relevant mass shell, since this is what determines the coupling to dileptons. Consequently, instead of the low momentum limit of the self energies, one is interested in their limit for large momentum $P \gg T$. Calculation shows that the ρ and a_1 self energies are each $\Pi^{\mu\nu} = (\delta^{\mu\nu} - P^\mu P^\nu/P^2)(g^2T^2/6)$, while the f_1 self energy is $\Pi^{\mu\nu} = \delta^{\mu\nu}(g^2T^2/3)$ at large $P \gg T$. Using $T_\chi^2 = 2\sigma_0^2$ and (3), in weak coupling at the critical temperature the pole masses in the vector meson propagators are given by

$$m_\rho^2(T_\chi) = m_{a_1}^2(T_\chi) = \frac{1}{3}\left(2m_\rho^2 + m_{a_1}^2\right) = (962\,MeV)^2 ,$$

$$m_{f_1}^2(T_\chi) = \frac{1}{3}\left(m_\rho^2 + 2m_{a_1}^2\right) = (1120\,MeV)^2 . \qquad (5)$$

On the right hand side of (5) and henceforth, whenever I write a mass such as m_ρ or m_{a_1}, implicitly I am referring to their values at zero temperature; any thermal pole mass is denoted by $m_\rho(T)$, *etc.*

Since in (2) the ω field does not interact with the scalar fields, the ω mass does not move, $m_\omega^2(T) = m_\omega^2$; $m_\omega^2(T)$ only shifts from effects of the anomaly. At the very least, it is apparent that the near degeneracy between the zero temperature masses of the ω and the ρ, and the a_1 and the f_1, is badly broken at nonzero temperature.

The width of the ρ can be computed by standard means [18]; at one loop order the only available mode is $\rho \to \pi\pi$. For a ρ decaying at rest,

$$\Gamma_\rho^\chi = \frac{g^2}{48\pi}\left(1 + 2n(m_\rho^\chi/2)\right)\frac{((m_\rho^\chi)^2 - 4(m_\pi^\chi)^2)^{3/2}}{(m_\rho^\chi)^2} . \quad (6)$$

Here $m_\pi^\chi = m_\pi(T_\chi)$ and $m_\rho^\chi = m_\rho(T_\chi)$ are the thermal pole masses at $T = T_\chi$, and $\Gamma_\rho^\chi = \Gamma_\rho(T_\chi)$. This is just the standard formula for the decay width of the ρ, except that there is a factor involving the Bose-Einstein distribution function, $n(E) = 1/(exp(E/T)-1)$, from stimulated pion emission in a thermal bath. At zero temperature, (6) gives a decay width that is about 20% too large, $\Gamma_\rho(0) \sim 179\,MeV$ instead of the experimental value of $150\,MeV$.

To obtain a somewhat realistic estimate of the width of the thermal ρ, the nonzero mass of the pion must be included. The full problem with $h \neq 0$ and $T \neq 0$ is rather complicated, since $m_\pi^\chi \sim T_\chi$. I adopt an approximate solution: the thermal effects are computed in the high temperature limit, including only the terms $\delta\mathcal{L} = (\lambda T^2/2)tr(|\Phi|^2) + (g^2T^2/12)((\vec{\rho}^\mu)^2 + (\vec{a}_1^\mu)^2)$. When $h \neq 0$ the definition of T_χ is ambiguous; I define T_χ as the point where $m_\sigma(T)$ has a minimum with respect to T. Doing so, for $m_\sigma = 600\,MeV$ I find $T_\chi = 226\,MeV$; at $T = T_\chi$, $f_\pi^\chi = 32\,MeV$, $m_\rho^\chi = 978\,MeV$, $m_{a_1}^\chi = 1002\,MeV$, $m_\pi^\chi = 185\,MeV$, $m_\sigma^\chi = 221\,MeV$, and $\Gamma_\rho^\chi = 278\,MeV$. For $m_\sigma = 1000\,MeV$ I find: $T_\chi = 221\,MeV$; at $T = T_\chi$, $f_\pi^\chi = 23\,MeV$, $m_\rho^\chi = 971\,MeV$, $m_{a_1}^\chi = 983\,MeV$, $m_\pi^\chi = 217\,MeV$, $m_\sigma^\chi = 263\,MeV$, and $\Gamma_\rho^\chi = 248\,MeV$. If I assume that the ρ width is too high by the same amount at T_χ as at $T = 0$, and so should be corrected by a factor of 150/179, I obtain $\Gamma_\rho^\chi = 233\,MeV$ for $m_\sigma = 600\,MeV$ and $\Gamma_\rho^\chi = 208\,MeV$ for $m_\sigma = 1000\,MeV$.

The form in which I have written (5) is a bit misleading, in that at leading order in weak coupling I can eliminate g entirely, to write expressions for the masses at T_χ solely in terms of the zero temperature masses. Nevertheless, it should be emphasized that this is a trick only of results to lowest order; the corrections to (5) and (6) are a power series in g^2 and λ, and so large. Thus the above numerical values are *not* meant to be taken as predictions, but *only* as suggestions of the magnitude of the possible effect. Perhaps, however, the *qualitative* features of a weak coupling analysis are reasonable. At zero temperature the splitting between the ρ and a_1 masses

are driven entirely by spontaneous symmetry breaking; it is sensible that the thermal fluctuations which restore the symmetry are of the same order as the shift upward in the (thermal) ρ mass. Similarly, while thermal broadening can be very significant if the ρ mass decreases, if the ρ mass increases these effects are naturally small, since then the π's are energetic, with momenta significantly larger than the temperature. One effect which I have neglected which increases Γ^χ_ρ is the thermal width of the π's; however, a more realistic value of T_χ is probably lower than the above [16], which lowers Γ^χ_ρ.

It is also of interest to compute the shift in the pole masses at low temperature. In the chiral limit we can make comparison with a general analysis of Eletsky and Ioffe [7], who show that the shift in the pole masses vanishes to order $\sim T^2$ about $T=0$. In gauged sigma models this holds for both the ρ and a_1 masses [5]. The first nonleading terms in the pole masses for the transverse fields are, in the chiral limit,

$$m^2_\rho(T) \sim m^2_\rho - \frac{g^2\pi^2 T^4}{45 m^2_\rho}\left(\frac{4m^2_{a_1}(3m^2_\rho+4p^2)}{(m^2_{a_1}-m^2_\rho)^2} - 3\right) + \dots \, ,$$

$$m^2_{a_1}(T) \sim m^2_{a_1} + \frac{g^2\pi^2 T^4}{45 m^2_\rho}\left(\frac{4m^2_{a_1}(3m^2_{a_1}+4p^2)}{(m^2_{a_1}-m^2_\rho)^2} + \frac{2m^4_\rho}{m^2_{a_1}(m^2_{a_1}-m^2_\sigma)} - \frac{m^2_{a_1}}{m^2_\rho}\right) + \dots \, , \tag{7}$$

where p^2 is the spatial momentum squared of the field. That is, while by the time of the chiral transition the thermal ρ mass goes up, and the a_1 mass down, about zero temperature they *start* out in the opposite direction: the ρ mass goes down, and the a_1 up!

Putting in the values of m_ρ, m_{a_1} and g, at zero momentum, $p=0$, I find that $(m^2_\rho(T)-m^2_\rho)/m^2_\rho = -(2.98\, T/m_\rho)^4$, while $(m^2_{a_1}(T)-m^2_{a_1})/m^2_{a_1} = +(3.16\, T/m_\rho)^4$ when $m_\sigma = 600\, MeV$, and $(m^2_{a_1}(T)-m^2_{a_1})/m^2_{a_1} = +(3.17\, T/m_\rho)^4$ for $m_\sigma = 1000\, MeV$. These values are interesting because the coefficients of T/m_ρ on the right hand side are relatively large: if we push them well beyond their range of validity, to $T\sim 200\, MeV$, they suggest that the shifts in the thermal ρ and a_1 masses can be significant, on the order of T_χ, as found in (5).

The shift in the thermal masses at low temperature can also be computed away from the chiral limit. When $m_\pi \neq 0$ I find that the ρ mass does not shift to $\sim T^2$, but the a_1 mass does,

$$m^2_{a_1}(T) \sim m^2_{a_1} + \frac{g^2 m^2_\pi T^2}{4 m^2_\sigma} + \dots \, . \tag{8}$$

As for the $\sim T^4$ term in the chiral limit, (7), when $m_\pi \neq 0$ the a_1 mass starts out by going *up* at low temperature. In QCD, except at the very lowest temperatures, this correction is small relative to that in (7): $(m^2_{a_1}(T)-m^2_{a_1})/m^2_{a_1} = +(.46\, T/m_\rho)^2$ for $m_\sigma = 600\, MeV$, and $(m^2_{a_1}(T)-m^2_{a_1})/m^2_{a_1} = +(.27\, T/m_\rho)^2$ for $m_\sigma = 1000\, MeV$.

I conclude by discussing the relationship with other approaches. By using a gauged linear sigma model for $T \leq T_\chi$, implicitly I am assuming that the behavior of QCD at nonzero temperature is dominated by the restoration of chiral symmetry, and not by deconfinement. This accords with current numerical simulations of lattice gauge theory [17], which indicates while there is no true phase transition in QCD, it lies close to a chiral critical point [19].

In contrast, if the phase transition were dominated by deconfinement, then as argued initially in [1], it is conceivable that the thermal ρ mass decreases with increasing temperature. For example, sum rule analyses of the phase transition can be construed as dominated by deconfinement. Generally, such analyses find that the thermal ρ mass goes down as T goes up [9] (see, however, [10]); about zero temperature, ref. [12] find that both the ρ and a_1 masses decrease to $\sim T^4$, contrary to (7). Using the experimental phase shifts, Shuryak and Thorsson [6] also find that the ρ mass decreases, by a small amount, at $T\sim T_\chi$.

Following Georgi, Brown and Rho, and others [8], have analyzed a sigma model where the ρ mass decreases monotonically with temperature. While their analysis uses a nonlinear sigma model, it can be reexpressed in terms of a linear sigma model. Assume that the explicit mass term for the gauge fields $\sim m^2$ in (1) vanishes, and that instead the local chiral symmetry is broken *only* by the term such as $\mathcal{L}_\kappa = \kappa\, tr(|\Phi|^2) tr((A^\mu_l)^2 + (A^\mu_r)^2)$, where κ is a dimensionless coupling constant. With such a term, up to T_χ the ρ mass *does* decrease uniformly with temperature; an easy calculation shows that in the chiral limit, $m^2_\rho(T_\chi) = m^2_{a_1}(T_\chi) = (2/3) m^2_\rho = (629\, MeV)^2$. However, setting $m=0$ and including $\mathcal{L}_\kappa$ manifestly violates the assumption of vector meson dominance, since then the local chiral symmetry is broken by a term with a dimensionless, instead of a dimensional, coupling constant.

In other words, which way the thermal ρ goes depends crucially upon whether or not vector meson dominance applies at nonzero temperature. If vector meson dominance holds, the thermal ρ mass goes up by T_χ; without vector meson dominance, there is no unique prediction.

To understand the relationship to the theory at $T \geq T_\chi$, it is necessary to remember that a constant feature of the lattice results [17] is that independent of the order of the phase transition, uniformly there appears to be a large increase in the entropy in a narrow region of temperature. Such a large increase in entropy cannot be described by the kind of gauged linear sigma models which I have been using. Consequently, I presume that such sigma models are valid *only* to a temperature just below T_χ, but *not* above.

In heavy ion collisions at ultrarelativistic energies,

then, if a mixed phase lives for a long time and dominates total dilepton production, a two state signal should appear in dilepton production. From the quark-gluon phase at $T = T_\chi^+$, dilepton production is dominated by the quark quasiparticles [20], presumably concentrated in a region below the zero temperature ρ peak. The hadronic phase at $T = T_\chi^-$ generates a thermal ρ peak; the position of this peak is model dependent, lying either above or below the zero temperature ρ peak, depending upon whether the assumptions of [4], [5], and this work, or those of [6], [8], [9], and [12], apply.

Whichever scenario applies, theoretically there are numerous indications that if it is possible to resolve relatively wide structure in dilepton production in ultrarelativistic heavy ion collisions — on the order of $T_\chi \sim 200\,MeV$ — then it might well reveal novel structure. While experimentally this is an *extremely* difficult task, the possible rewards appear well worth the effort.

I happily (if belatedly) acknowledge that an inspirational colloquium on the quark-gluon plasma by W. J. Willis at Yale University in 1981 originally [1] stimulated my interest in this problem. During the present investigation I benefited from discussions with J. Bijnens, V. Eletsky, T. Hatsuda, S.-H. Lee, M. Rho, E. Shuryak, A. Sirlin, C. Song, L. Trueman, A. Weldon, and especially S. Gavin. This work is supported by a DOE grant at Brookhaven National Laboratory, DE-AC02-76CH00016.

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[13] Besides the mass term in (1), there is also the isosinglet invariant $\widetilde{m}^2((tr(A_l^\mu))^2 + (tr(A_r^\mu))^2)/4$; $m_\omega^2 = m_\rho^2 + \widetilde{m}^2$ and $m_{f_1}^2 = m_{a_1}^2 + \widetilde{m}^2$. Experimentally, however, $\widetilde{m}^2/m^2$ is *very* small: $\widetilde{m}^2/m^2 \sim .03$ from $m_\omega = 782\,MeV$, $\widetilde{m}^2/m^2 \sim .11$ from $m_{f_1} = 1285\,MeV$; $\widetilde{m}^2/m^2$ is $\sim 1/N_c$ in the limit of a large number of colors, $N_c \rightarrow \infty$ (J. Bijnens, private communication).

[14] I note that vector meson dominance is consistent with the limit of a large number of colors, $N_c \rightarrow \infty$. Vector meson dominance implies $j_{em}^\mu \sim (m^2/g)\rho^\mu$; assuming that $m \sim 1$ and $g \sim 1/\sqrt{N_c}$, $\langle j_{em}^\mu j_{em}^\nu \rangle \sim N_c$, as follows from the underlying quark diagrams. Consider corrections in $1/N_c$: the ρ peak, which is of order $\sim N_c$ in height, acquires a width $\sim 1/N_c$. At this order $\pi^+\pi^-$ production also contributes; since hadrons are free at large N_c, this is calculable, of order ~ 1. Thus the area under both the ρ peak and the $\pi^+\pi^-$ continuum are of order ~ 1.

[15] Whether $m_\sigma = 600\,MeV$ or $1000\,MeV$ is uncertain; see C. Michael, Liverpool preprint LTH340, hep-lat/9412032 and G. Janssen, B. C. Pearce, K. Holinde, and J. Speth, Julich preprint KFA-IKP-TH-1994-40, nucl-th/9411021. In QCD the value of the ratio m_σ/m_ρ is uncertain, since both the ρ and especially the σ have large widths. I make the trivial remark that the value of m_σ/m_ρ *is* uniquely defined and physically sensible in either the quenched approximation or for large N_c, since in both limits, mesons don't decay.

[16] Because of the factor of m_{a_1}/m_ρ which enters into f_π in (4), this is significantly higher than the estimate of T_χ without gauge fields, which is just $T_\chi = \sqrt{2} f_\pi = 131\,MeV$; it is also significantly higher than the estimate from lattice simulations, which are $T_\chi \sim 150\,MeV$ [17]. I presume this is a shortcoming of the gauged linear sigma model.

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